

Internship and thesis proposal

Laboratory name: **PHENIX** (UMR 8234)

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Internship location: **Sorbonne Université, place Jussieu, Paris 5ème**

Thesis possibility after internship: **YES**

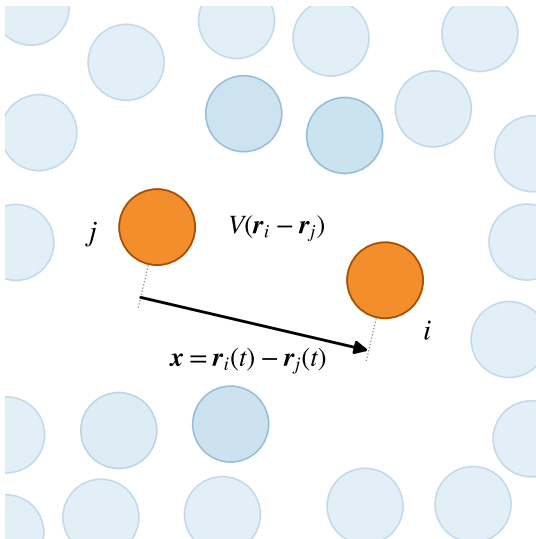
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Rare encounters in a crowd: fluctuations of pair correlations in liquids

The Dean–Kawasaki equation [1, 2] is an exact stochastic equation for the density field of a suspension of interacting Brownian particles. Over the past decade, it has become the cornerstone of stochastic density functional theory, a versatile framework to compute dynamical correlations and transport coefficients in soft matter, from tracer diffusion in crowded media to the conductivity of electrolytes [3, 4]. In practice, however, the equation is almost always linearized around a homogeneous state. This approximation is accurate only for weak interactions and high densities, and it washes out the local structure of the suspension, which the theory of liquids encodes in pair-level quantities such as the pair correlation function $g(r)$ [5].

The goal of this internship is to extend Dean’s approach to pair-level quantities, and to write and solve stochastic equations for them rather than for the density itself. A natural object is the microscopic pair density $\rho_{ij}(\mathbf{x}, t) = \delta(\mathbf{x} - [\mathbf{r}_i(t) - \mathbf{r}_j(t)])$, whose average over all pairs yields the pair correlation function: $\langle \sum_{i \neq j} \rho_{ij}(\mathbf{x}, t) \rangle = N \rho_0 g(\mathbf{x})$ in a homogeneous system of N particles at density ρ_0 . Its dynamics can be derived exactly from the Langevin equations of the particles, and it naturally gives rise to stochastic versions of classical objects of liquid-state theory. A prominent example is the cavity distribution function $y(r) = e^{\beta V(r)} g(r)$, which removes the direct contribution of the pair potential from g : it remains smooth even for singular, hardcore interactions, and it is the central object of the diagrammatic expansions and integral-equation closures (Percus–Yevick, hypernetted chain) that underlie the quantitative description of dense liquids [5]. Unlike the density, these quantities retain information about the interactions at short distances, and they offer a new route to go beyond mean field.

At equilibrium, the instantaneous distribution of the separation between two particles is entirely determined by g . Its dynamics, however, contains much richer information. A more ambitious goal is



In a suspension of Brownian particles interacting through the pair potential V , we focus on the stochastic dynamics of the separation $\mathbf{x} = \mathbf{r}_i(t) - \mathbf{r}_j(t)$ between two particles, described by the microscopic pair density $\rho_{ij}(\mathbf{x}, t)$. Its average yields the pair correlation function g , while its full dynamical statistics characterizes rare events, such as two particles remaining abnormally close over long times.

to characterize the statistics of time-integrated pair observables, such as the fraction of time two particles spend close to each other, or the empirical pair distribution measured over a long time window. Path-integral methods, which were recently used to compute non-Gaussian density fluctuations from the Dean–Kawasaki equation [6], provide a natural framework to do so. Applied to the equations obeyed by pair quantities, they could give access to the corresponding large deviation functions, i.e. to the probability of rare events, such as two particles remaining abnormally close or far apart over long times. Such results would extend naturally to nonequilibrium suspensions, where g itself is not known.

The intern will first derive the stochastic equations for pair quantities and examine their structure. They will then set up the path-integral formulation and identify controlled approximations, guided by the classical closures of the theory of liquids. Predictions will be compared with Brownian dynamics simulations. The work will be mostly analytical (stochastic calculus, path integrals, large deviations, theory of liquids), and is well suited to a student who enjoys fundamental questions in statistical physics.

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- [6] L. Le Bon, A. Carof, P. Illien, *Phys. Rev. E* **112**, 034127 (2025). [arXiv:2501.16206](#).