

Internship and thesis proposal

Laboratory name: **PHENIX** (UMR 8234)

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Internship location: **Sorbonne Université, place Jussieu, Paris 5ème**

Thesis possibility after internship: **YES**

Funding: **NO**

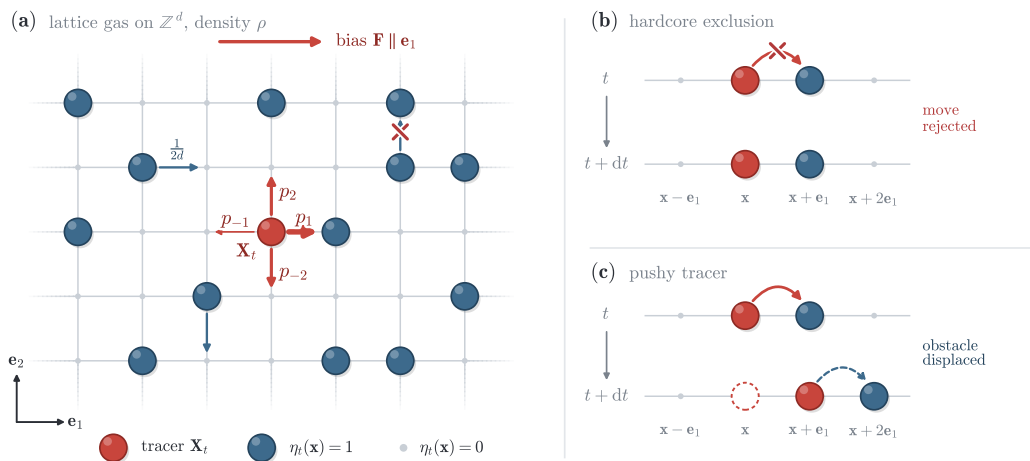
Microscopic dynamics of bullies

Characterizing the motion of individuals in complex and crowded environments is of utmost importance for understanding the emergence of macroscopic kinetic behavior. Whether it be the erratic exploration of mazes by clueless ants [1], the driven dynamics of a probe immersed in complex biological media during a microrheology experiment [2], or the constrained trajectory of a shopper on a busy day, the same scenario repeats across scales. More precisely, the local dynamical rules dictate the long-term macroscopic behavior: how long it takes for the ants to escape, how fast the probe travels, or how likely it is for the shopper to reach their destination.

Lattice gas models [3, 4] provide a fruitful theoretical framework to analytically investigate these various situations. Typically, the lattice gas consists of N particles on $\mathbb{Z}^d$ performing independent random walks. The interaction mechanism is hardcore exclusion: no two particles can occupy the same lattice site at the same time, and an attempted move onto an occupied site fails. To characterize the particle-environment interaction, it is convenient to choose a tracer particle $\mathbf{X}$, give it a drift, and study its statistical properties. In particular, in the large- N thermodynamic limit, the dependence of the cumulants of the tracer position on the effective particle density ρ , such as the speed v or the diffusivity D ,

$$\langle \mathbf{X}_t \rangle \underset{t \rightarrow \infty}{\sim} v(\rho) t, \quad \langle \mathbf{X}_t^2 \rangle - \langle \mathbf{X}_t \rangle^2 \underset{t \rightarrow \infty}{\sim} D(\rho) t,$$

has been extensively studied and displays rich behavior.



(a) Lattice gas on $\mathbb{Z}^2$. The tracer particle (in red) performs a driven asymmetric random walk among “free” blue particles. The density ρ of particles, as well as the relative probabilities $p_{1/-1}$, dictate the long-time dynamics of the tracer, in particular the inhomogeneous density profiles surrounding it. (b) Hardcore exclusion prevents the tracer from moving onto a site occupied by a free gas particle: no motion occurs. (c) Pushy dynamics allow the tracer to boss gas particles around, enhancing its mobility.

Oftentimes, however, the hardcore interaction is unrealistic; it seems much easier to navigate a crowd by consistently shoving small obstacles! From a modeling viewpoint, this amounts to allowing the tracer to displace another particle (see figure) when attempting a move previously forbidden by pure exclusion. Such “pushy” dynamics have received close to no attention [5]. In particular, how the speed and diffusivity are impacted by these pushing events remains a fully open question. Investigating this is the main goal of the internship, and we expect that the tools developed will lead to a more general characterization of what the local environment looks like in the moving frame of the tracer.

The internship combines both numerical and analytical approaches. While numerical methods for lattice gases are by now standard, they need to be adapted to the pushy exclusion mechanism. From the analytical viewpoint, we will adapt the tools developed at length in [6]. Specifically, we will first derive the master equation governing the dynamics of the probability density $P(\mathbf{X}, \eta; t)$ of the time-dependent configuration $(\mathbf{X}_t, \eta_t)$, where $\eta_t \in \{0, 1\}^{\mathbb{Z}^d}$ is the dynamical lattice occupancy vector, with $\eta_t(\mathbf{x})$ accounting for the presence of a particle at a given site $\mathbf{x}$. In turn, we expect that this equation will allow for the analysis of the various cumulants of $\mathbf{X}_t$, as well as of the correlations between $\mathbf{X}$ and η (at least in the high- and low-density regimes). These are key to analyzing the emerging drift and diffusivity of the tracer, and to answering the one important question: is it worth being a bully?

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