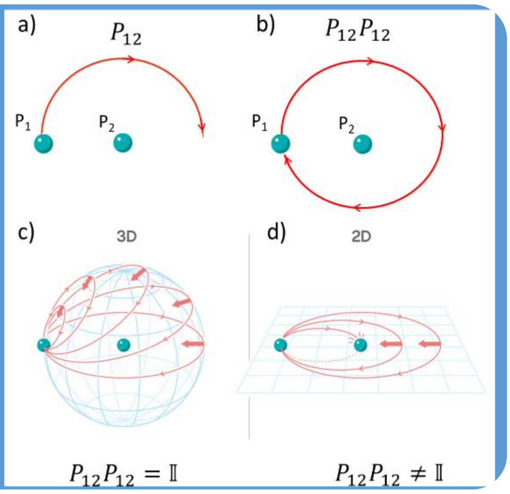
Anyon statistics in fractional quantum Hall conductors

I. Context of the project

Exchange statistics are related to the phase φ accumulated by the wavefunction describing the state of an ensemble of undistinguishable particles when two particles are exchanged. In the three-dimensional world, particles are divided between bosons, that obey $\varphi = 0$ and tend to bunch together, and fermions, for which $\varphi = \pi$ and that exclude each other via the Pauli exclusion principle. The situation is completely different in two-dimensional systems, which allow the existence of quasiparticles with intermediate statistics between fermions and bosons, leading to intermediate degrees of bunching and exclusion. As their exchange phase can take any value, these quasiparticles have been called anyons (see box 1). Interestingly these quasiparticles keep a memory of the number of exchanges between them, which is protected from local perturbations of the anyons trajectories: one speaks of topological protection. This protection is at the heart of the current interest for anyons, as specific types of anyons, called non-abelian, are the building blocks of topological quantum computing that would be protected from decoherence.

Box 1: Anyons

a) Consider two particles P_1 and P_2 represented as green balls with hardcore repulsion potential. Moving particle 1 around particle 2 along a half circle corresponds to the exchange operation P_{12} of the two particles. b) A full circle corresponds to twice the exchange operation $P_{12}P_{12}$, the first particle goes back to its initial position. c) In 3D, $P_{12}P_{12} = \mathbb{I}$, the identity operator, meaning that $P_{12} = e^{i\varphi}\mathbb{I}$ with $\varphi = 0$ (bosons) or $\varphi = \pi$ (fermions). It is deduced from topological arguments: φ should be a property of the particles and not depend on the specific path taken by particle 1. In 3D, this path (circle) can be continuously deformed and shrink to a point, imposing $P_{12}P_{12} = \mathbb{I}$. d) This possibility does not exist in 2D where P_1 cannot move through P_2. The constraint $P_{12}P_{12} = \mathbb{I}$ is thus lifted in 2D and φ can take any value. These particles have thus been called anyons.



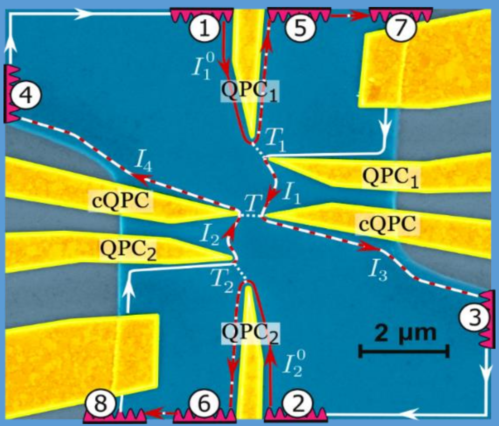
The strongly correlated phases of the fractional quantum Hall (FQH) effect have been predicted to host anyons carrying a fractional charge and obeying fractional statistics. The fractional charge has been observed twenty years ago by partitioning a beam of anyons and by measuring the resulting current noise, which is proportional to the fractional charge. Despite numerous attempts, no direct evidence of fractional statistics had been obtained until two experiments provided the first observations of fractional statistics in 2020. In particular, extending noise measurements in the geometry of an anyon collider, our team at LPENS in collaboration with our partners from C2N demonstrated the fractional statistics of anyons at the filling factor 1/3 of the FQH effect. The purpose of this internship and PhD is to extensively study the properties of anyons for different phases of the FQH effect using the geometry of the anyon collider.

I. Objectives of the internship and PhD

The first and main objective of the internship/Phd project is to quantitatively study the fractional statistics of anyons for different topological orders (controlled by the filling factor) in the anyon collider geometry. Different phases of the FQH effect obtained by tuning the value of the magnetic field will be studied. For the Laughlin states (filling factor $\nu = 1/m$), the topological order of the bulk conductor is fully characterized by the single number m imposing the Hall conductance $G = e^2/(hm)$. This implies both a simple structure of the edge with a single edge channel and a simple relation between the fractional charge e/m and the exchange phase π/m. More complex topological orders, governed by several numbers, are obtained for FQH states belonging to the Jain sequence (e.g.

$\nu = 2/5$ and $\nu = 2/3$). It results in a more complex structure of the edges with several edge channels (two for $\nu = 2/5$ and $\nu = 2/3$). In this richer situation, Hall conductance, charge and statistics are no longer related by $G/(e^2/h) = e^*/e = \varphi/\pi$. The first experimental measurements of anyon statistics obtained for $\nu = 2/5$ show a clear difference with the $\nu = 1/3$ case but are not fully understood yet, due to the more complex edge structure in this case.

Box 2: The anyon collider



Sample fabricated at C2N: current propagates along the edge channels (red lines) of the two-dimensional electron gas (in blue). Anyons are randomly emitted by partitioning the input currents I_1^0 and I_2^0 with transmissions T_1 and T_2 using metallic split gates (in gold) called quantum point contacts (QPC1 and QPC2). Anyons collide on the central quantum point contact cQPC used as a beam splitter with transmission T. Measuring the cross-correlations between the output currents I_3 and I_4 probes the tendency of particles to bunch together in one output or to exclude each other in different outputs depending on the fractional statistics.

The most challenging part of the project will be the investigation of non-abelian anyons (which are relevant for topological quantum computing) at the filling factor 5/2. The non-abelian nature of the ground state has recently been established by thermal transport measurements, but the exact nature of the ground state is not yet settled and direct signatures of the statistics are still elusive. A few experimental results in interferometry experiments have been obtained and are consistent with non-abelian statistics but there is a consensus that these experiments require confirmations, which we plan to provide using the different approach of the collider.

The second objective of the project will consist in understanding the role of decoherence and relaxation on experimental signatures of anyonic statistics. Surprisingly, even though fractional statistics were predicted 40 years ago, it took a long time to obtain direct evidences, suggesting that these signatures are fragile, which also questions the concept of topological protection. We will investigate the role of temperature and relaxation which are known to be important in the context of electron collisions. These effects, governed by the distance between input QPC's and the central beam-splitter will be studied here both experimentally and theoretically for anyon collisions. These studies will be particularly relevant when the edge structure is complex (e.g. for $\nu = 2/5$, $2/3$ and $5/2$), allowing for strong interactions between neighboring edge channels.

The student will take part to low temperature experiments in a dry dilution fridge at a temperature of ~15 mK and under high magnetic fields to reach the fractional quantum Hall regime. Experimental techniques will combine conductance and noise measurements at low and high (GHz) frequencies.

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