

INTERNSHIP PROPOSAL

Laboratory name: Institut d'Astrophysique de Paris

CNRS identification code: UMR 7095

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Internship location: Institut d'Astrophysique de Paris, CNRS & Sorbonne Université

Thesis possibility after internship: YES

Funding: NO [not guaranteed] but application possible to the ED127 Doctoral School

Small-scale chaos and the Vlasov limit of N-body dynamics

Context

Gravity is a long-range interaction. As a result, each star in a stellar system interacts with all the others. In the limit of a large number, N , of particles, the exact (Klimontovich) dynamics of the N particles is expected to converge to the smooth (Vlasov) dynamics of a continuous distribution function [1]. Yet, because gravity diverges on small scales, the available rigorous proofs of this convergence are typically limited to interactions smooth below some softening scale [2-4]. Indeed, in an N -body system, close encounters drive an exponential divergence of nearby trajectories. Such a small-scale chaos can be tracked via a Lyapunov time characterising the exponential separation of nearby realisations. Strikingly, in the absence of softening, for 3D self-gravitating systems, the Lyapunov time decreases with N [5-7]. Conversely, for a fixed softening, for N large enough, the Lyapunov time grows with N [8-10].

Upshot

This internship focuses on the role of small-scale chaos in the convergence of the Klimontovich equation towards the Vlasov one. For that purpose, we will investigate homogeneous multi-periodic systems of N particles in dimension D [11,12]. Particles will interact through a pairwise potential given in Fourier space by $-G/|k|^a$, with " a " setting the strength of the small-scale divergence ($a=-2$ is gravity). Softening will be mimicked by setting the interaction to zero for $|k| > k_{\max}$. Using non-uniform fast Fourier transforms [13] to integrate the equations of motion and their variational extensions [14,15], as well developing analytical predictions, we will measure and predict Lyapunov times as a function of $(N, D, a, k_{\max})$. We will identify the mechanism that controls the transition between the different regimes of chaos. Ultimately, this internship should offer new clues on the microscopic foundations of collisionless dynamics, as well as on the impact of softening on N -body simulations.

References

- [1] Campa+, 2014, Oxford Press
- [2] Braun+, 1977, CMP, 56, 101
- [3] Lazarovici+, 2017, ARMA, 225, 1201
- [4] Duerinckx+, 2026, 2608.04104
- [5] Goodman+, 1993, ApJ, 415, 715
- [6] Hemsendorf+, 2002, ApJ, 580, 606
- [7] Portegies Zwart+, 2022, A&A, 659, A86
- [8] Asano+, 2026, A&A, 712, A233
- [9] Di Cintio+, 2019, MNRAS, 489, 5876
- [10] Latora+, 1998, PRL., 80, 692
- [11] Weinberg, 1993, ApJ, 410, 543
- [12] Magorrian, 2021, MNRAS, 507, 4840
- [13] Greengard+, 2004, SIAM, 46, 443
- [14] Rein+, 2016, MNRAS, 459, 2275
- [15] Skokos+, 2010, PRE, 82, 036704

Requirement

Strong interest in theoretical astronomy, dynamics, analytical and numerical work. For full consideration, prospective students should send their CV, transcripts, and a (short) cover letter to fouvry@iap.fr, before Nov. 15, 2026.

Framework

The internship will be supervised by Jean-Baptiste Fouvry (IAP, Paris) within the ANR BEYOND-BL.